APRIL QUANTUM COMPUTING MEETUP

- Agenda
  - Introductions and Thanks to our Host Bank Of America
  - Food/Pizza
  - Short Course on Quantum Mechanics
  - IBM Presentation – Quantum Computing, Grover Search and the IBM Quantum Experience
  - Deep dive into the Math for Quantum Computing by way of two Iterations of Grover Search
- Presentations can be found at github.com/quantumnewyork/Meetings
- Next meeting either May 4, May 11
- Looking for hosts, presenters, topics, suggestions

QUANTUM COMPUTING : SUPERPOSITION AND INTERFERENCE

A LOOK AT A QUANTUM ALGORITHM CALLED GROVER SEARCH

LESSON PLAN

- Description of Grover Search
- Introduction to Superposition and Interference in Quantum Mechanics through experiments
- Exposition and Demo of how Superposition and Interference are used in Quantum Computing, focusing on Grover Search.
- A worked mathematical example of Grover Search with three Q-bits

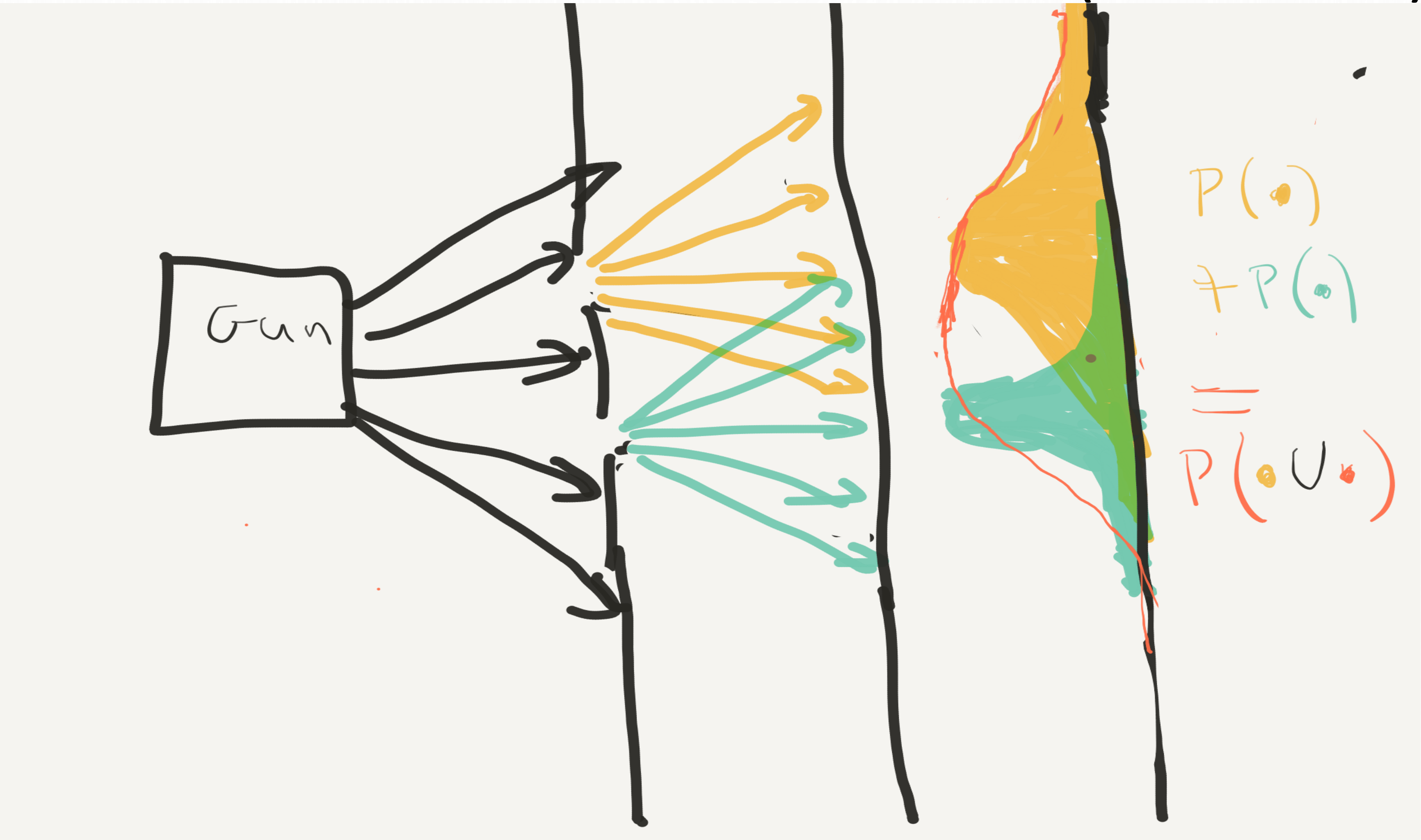
GROVER SEARCH

- Given a Phone Book Containing N unsorted entries (or any list of Unstructured Data)
- Find an entry that corresponds to a particular name (or criteria)
- Returns a most probable solution
- Leverages Quantum Mechanics(QM) Properties
  - Superposition
  - Particle/Wave Duality and Interference
  - Coherence and Decoherence

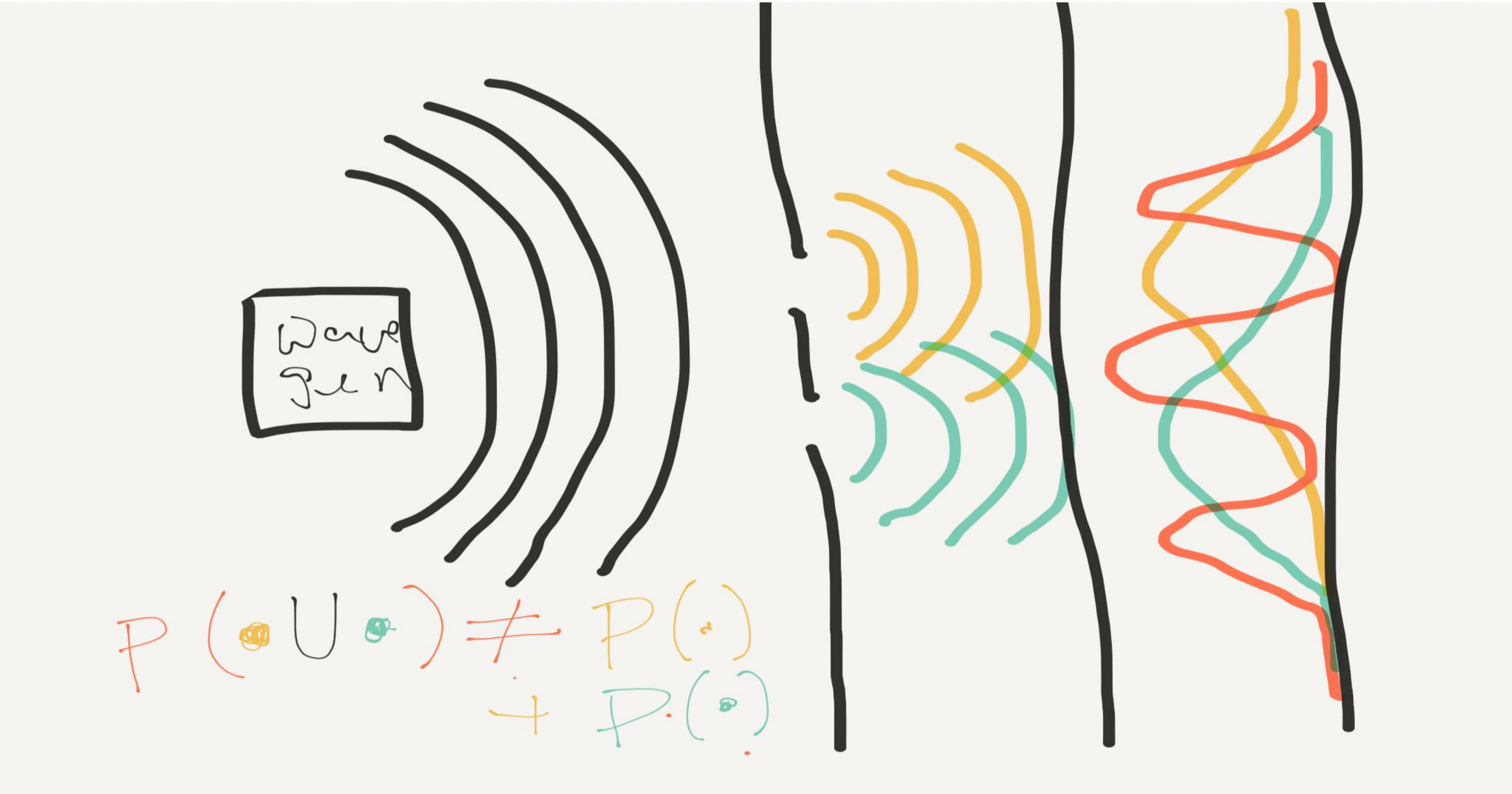
REVIEW OF QM PROPERTIES

- Wave/Particle Duality : Particles can Exhibit Properties of Waves
- Superposition : Particles can be in many states at once
- Interference : Wave/Particles in Superposition can interfere
- Coherence/Decohence : Observers collapse Superposition to an individual state
- Observed result is probabilistic

A CLASSICAL PARTICLE EXPERIMENT (FEYNMAN)



A CLASSICAL WAVE EXPERIMENT (FEYNMAN)



A CLASSICAL SUMMARY

- Particles impacts exhibit predictable additive properties irrespective of how many openings in our experiment
- Particle systems can be modeled with standard probability theory (additive in the or case)
- Waves exhibit interference and are very sensitive to how many openings are in the experiment
- Require a lot more math to model and are not additive.

A QUANTUM EXPERIMENT (DOUBLE SLIT)

- The photon seems to behave like a particle with one slit
- With two slits, it seems to be going through both openings – Superposition
- It interferes with itself as if it was a wave – Wave/Particle Duality
- If you observe the openings it will seem like there is only one slit open – Decoherence

QM PROPERTIES SUMMARY

- So if we believe in QM … what does this mean to me ??
  - “Naturally Occuring” physical phenomenon for “many simultaneous states” representation
  - “Many simultaneous states” can be constructed to generate “Outcomes”
  - “Outcomes” can be observed repeatedly (possibly influencing probabilistically a state) to generate a most probable “Result”
- So how do I go about leveraging QM to Generate Outcomes?? At the quantum level, we need to:
  - Define States (ie openings in our Slit)
  - Define Wave functions of each state
  - Define a way to combine wave functions to represent interference
  - Define a way to transform waves (amplify/dampen) without observing them
  - Define an observation device
- This is what Quantum Computing is about….

EXPOSITION AND DEMO OF HOW SUPERPOSITION AND INTERFERENCE ARE USED IN QUANTUM COMPUTING, FOCUSING ON GROVER SEARCH.

- Welcome IBM

A WORKED MATHEMATICAL EXAMPLE OF GROVER SEARCH WITH THREE Q-BITS

- See hand written notes

①

BRA-KET NOTATION

$$V = \begin{bmatrix} V_0 \\ \vdots \\ V_n \end{bmatrix} = |V\rangle \quad : \text{ket-}V$$

$$\bar{V}^T = \langle V| = [\bar{V}_0 \cdots \bar{V}_n]$$

where $V_i = a + bi$
$\bar{V}_i = a - bi$: complex conjugate

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inner Product

$$\langle U|V\rangle = \bar{U}^T V = [\bar{U}_0 \cdots \bar{U}_n] \begin{bmatrix} V_0 \\ \vdots \\ V_n \end{bmatrix}$$

$$U_i = a_i + ib_i$$

$$V_i = c_i + id_i$$

$$\langle U|V\rangle = \bar{U}_0 V_0 + \cdots + \bar{U}_n V_n$$

$$= (a_0 - ib_0)(c_0 - id_0) + \cdots + (a_n - ib_n)(c_n + id_n)$$

$$= [(a_0 c_0 - b_0 d_0) - i(b_0 c_0 + a_0 d_0)] + \cdots + [(a_n c_n - b_n d_n) - i(b_n c_n + a_n d_n)]$$

②

$$\langle v|v\rangle = [\bar{v}_0 \cdots \bar{v}_n]\begin{bmatrix} v_0 \\ \vdots \\ v_n \end{bmatrix} \qquad v_i = a_i + i b_i$$

$$= [(a_0^2+b_0)^2 - i(a_0 b_0 - a_0 b_0)] + \cdots + [(a_n^2+b_n^2) - i(a_0 b_0 - a_0 b_0)]$$

$$= \sum a_i^2 + \sum b_i^2 \geq 0$$

only 0 if $a_i = 0$ and $b_i = 0$.

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outer product

$$|v\rangle\langle u| = \begin{bmatrix} v_0 \\ \vdots \\ v_n \end{bmatrix} [\bar{u}_0 \cdots \bar{u}_m] = \begin{bmatrix} v_0\bar{u}_0 & v_0\bar{u}_1 & \cdots & v_0\bar{u}_m \\ \vdots & & & \\ v_n\bar{u}_0 & \cdots & & v_n\bar{u}_m \end{bmatrix}$$

$$(|v\rangle\langle u|)|w\rangle = |v\rangle\underline{\langle u|w\rangle} = \underline{\langle u|w\rangle}|v\rangle$$

Scalar

tensor product

$$|u\rangle|v\rangle = |uv\rangle = \begin{bmatrix} u_0 \\ \vdots \\ u_m \end{bmatrix} \otimes \begin{bmatrix} v_0 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} u_0v_0 \\ \vdots \\ u_mv_n \end{bmatrix}$$

$$|u\rangle|u\rangle = |u\rangle^{\otimes 2} = \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} \otimes \begin{bmatrix} u_0 \\ u_1 \end{bmatrix} = \begin{bmatrix} u_0^2 \\ u_0u_1 \\ u_1u_0 \\ u_1^2 \end{bmatrix}$$

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we Define "special" u and v of a qubit

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

computational Basis

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\langle 0|0\rangle = \langle 1|1\rangle = 1$

$\langle 0|1\rangle = \langle 1|0\rangle = 0$

and Quantum State vector

$|\psi\rangle = a_0|0\rangle + a_1|1\rangle,$

norm of $|\psi| = \sqrt{\langle\psi|\psi\rangle} = 1$

where $a_0 = a_0 + ib_0$

$a_1 = a_1 + ib_1$

$= |a_0|^2 + |a_1|^2 = 1$

a_0, a_1 is amplitude of

measure $|0\rangle$ and $|1\rangle$

why does a_0, a_1 need to be complex. (4)

Lets go back to bullets and waves

classical prob : 2openings

P(Opening 1) = 50%

P(opening 2) = 50%

P(op1 or op2) = P(op1) + P(Op2)
= 100%.

But Quantum → waves above doesnt hold.

P(opening 1) : $a_0 + b_0 i$: call a_0 :

P(opening 2) : $a_1 + b_1 i$: call a_1

P(opening 1) defined as $|a_0|^2 : a_0\bar{a}_0$

notation $(a_0 + ib_0)(a_0 - ib_0)$

$= a_0^2 + b_0^2$

P(opening 2) : defined as $|a_1|^2 : a_1\bar{a}_1$

$= a_1^2 + b_1^2$

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$\psi = a_0|0\rangle + a_1|1\rangle$

$p(op1) = |a_0|^2 \quad : \quad a_0^2 + b_0^2$

$p(op2) = |a_1|^2 \quad : \quad a_1^2 + b_1^2$

$p(op1 + op2) = |a_0 + a_1|^2$

$$= \big((a_0+a_1) + i(b_0+b_1)\big)\big((a_0+a_1) - i(b_0+b_1)\big)$$

$$= \overbrace{(a_0^2+b_0^2)}^{|a_0|^2} + \overbrace{(a_1^2+b_1^2)}^{|a_1|^2} + \underbrace{2(a_0a_1 + b_0b_1)}_{\text{interference}} \; *$$

more probabilities

$\psi = a_0|0\rangle + a_1|1\rangle$

$\langle\psi|\psi\rangle = 1$

$$\underbrace{(\bar{a}_0\langle 0| + \bar{a}_1\langle 1|)}_{\psi^\dagger}\underbrace{(a_0|0\rangle + a_1|1\rangle)}_{\psi}$$

$$= \bar{a}_0 a_0\langle 0|1\rangle + \bar{a}_0 a_1 \underbrace{\langle 0|1\rangle}_{0} + \bar{a}_1 a_0 \underbrace{\langle 1|0\rangle}_{0} + a_1\bar{a}_1\langle 1|1\rangle$$

$$= \bar{a}_0 a_0 + a_1\bar{a}_1 = |a_0|^2 + |a_1|^2 = 1 \; ✓$$

Examples of states (6)

$\psi_1 = a_0|0\rangle + a_1|1\rangle \qquad |0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \quad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

$\psi_2 = a_0|000\rangle + a_1|001\rangle + a_2|010\rangle + \cdots + a_7|111\rangle$

$$\psi_2 = a_0\begin{bmatrix}1\\ \vdots\\0\end{bmatrix} + a_1\begin{bmatrix}0\\1\\ \vdots\\0\end{bmatrix} + \cdots + a_7\begin{bmatrix}0\\0\\ \vdots\\1\end{bmatrix}$$

Some operations "gates" : Hadamard : fair flip

$$H: \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)\langle 0| + \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)\langle 1|$$

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}\underbrace{\langle 0|0\rangle}_{1} + \frac{|0\rangle - |1\rangle}{\sqrt{2}}\underbrace{\langle 1|0\rangle}_{0}$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

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inner Product

$$\langle U|V\rangle = \overline{U}^T V = [\overline{U}_0 \cdots \overline{U}_n] \begin{bmatrix} V_0 \\ \vdots \\ V_n \end{bmatrix}$$

$U_i = a_i + ib_i$

$V_i = c_i + id_i$

$$\langle U|V\rangle = \overline{U}_0 V_0 + \cdots + \overline{U}_n V_n$$

$$= (a_0 - ib_0)(c_0 - id_0) + \cdots + (a_n - ib_n)(c_n + id_n)$$

$$= [(a_0c_0 - b_0d_0) - i(b_0c_0 + a_0d_0)] + \cdots + [(a_nc_n - b_nd_n) - i(b_nc_n + a_nd_n)]$$

$$H|1\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}\langle 0|1\rangle + \frac{|0\rangle - |1\rangle}{\sqrt{2}}\langle 1|1\rangle$$

$$= \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

so: $H|0\rangle = \underbrace{\left(\frac{1}{\sqrt{2}}\right)}_{a_0}|0\rangle + \underbrace{\left(\frac{1}{\sqrt{2}}\right)}_{a_1}|1\rangle$ $\quad |a_0|^2 = \frac{1}{2}$, $|a_1|^2 = \frac{1}{2}$

$H|1\rangle = \underbrace{\left(\frac{1}{\sqrt{2}}\right)}_{a_0}|0\rangle + \underbrace{\left(-\frac{1}{\sqrt{2}}\right)}_{a_1}|1\rangle$

given a system in either state $|0\rangle$ a $|1\rangle$ produces equal prob

Some Examples

$$H^2|00\rangle = H|0 \otimes H|0$$

$$= \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}} =$$

$$= \frac{1}{\sqrt{4}}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

$|X\rangle = a_0|000\rangle + a_1|001\rangle + \cdots + a_7|111\rangle$

init state $1|000\rangle$

$H^3|000\rangle = \frac{1}{2\sqrt{2}}|000\rangle + \cdots + \frac{1}{2\sqrt{2}}|111\rangle = \sum|x\rangle$
$= \psi$

$a_i = \frac{1}{2\sqrt{2}}$

we know the right answer is $|011\rangle$.

The goal is to amplify $\underline{\underline{a_3}}$.

step 1: send ψ into oracle. phase shift

$\Rightarrow X_{S1} = \frac{1}{2\sqrt{2}}|000\rangle + \frac{1}{2\sqrt{2}}|001\rangle + \frac{1}{2\sqrt{2}}|010\rangle - \frac{1}{2\sqrt{2}}|011\rangle$

$+ \cdots + \frac{1}{2\sqrt{2}}|111\rangle$

step 2: perform diffusion transform

$[2|\psi\rangle\langle\psi| - I]|X_{S1}\rangle$

$= [2|\psi\rangle\langle\psi| - I]\left[|\psi\rangle - \frac{2}{2\sqrt{2}}|011\rangle\right]$

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$$= 2|\psi\rangle\langle\psi|\psi\rangle - \frac{\cancel{2}\cdot 2}{\cancel{2}\sqrt{2}}|\psi\rangle\langle\psi|011\rangle - \psi + \frac{2}{2\sqrt{2}}|011\rangle$$

$\langle\psi|\psi\rangle = 1$

$$= 2|\psi\rangle(1) - \psi - \frac{2}{\sqrt{2}}|\psi\rangle\langle\psi|011\rangle + \frac{1}{\sqrt{2}}|011\rangle$$

$$= \psi - \frac{2}{\sqrt{2}}|\psi\rangle\langle\psi|011\rangle + \frac{1}{\sqrt{2}}|011\rangle$$

$$\langle\psi|011\rangle = \langle 011|\psi\rangle$$

$$\langle 011|\psi\rangle = \frac{1}{2\sqrt{2}}\underbrace{\langle 011|000\rangle}_{0} + \cdots + \frac{1}{2\sqrt{2}}\underbrace{\langle 011|011\rangle}_{1} + \cdots \frac{1}{2\sqrt{2}}\underbrace{\langle 011|111\rangle}_{0}$$

$$= \frac{1}{2\sqrt{2}}$$

$$= \psi - \frac{\cancel{2}}{\sqrt{2}}|\psi\rangle \cdot \frac{1}{\cancel{2}\sqrt{2}} + \frac{1}{\sqrt{2}}|011\rangle = \frac{1}{2}\psi + \frac{1}{\sqrt{2}}|011\rangle$$

$$\frac{1}{2}\varphi + \frac{1}{\sqrt{2}}|011\rangle = \qquad \alpha \frac{1}{2\sqrt{2}} \sum_{0}^{7} |x\rangle$$

$$\frac{1}{4\sqrt{2}}|000\rangle + \frac{1}{4\sqrt{2}}|001\rangle + \left(\frac{1}{4\sqrt{2}}|011\rangle + \frac{1}{\sqrt{2}}|011\rangle\right)$$

$$+ \cdots + \frac{1}{4\sqrt{2}}|111\rangle$$

$$= \frac{1}{4\sqrt{2}}|000\rangle + \cdots + \left(\frac{5}{4\sqrt{2}}|011\rangle\right) = \varphi_{new}$$

Repeat on φ_{new} after next iteration

$a_3 |011\rangle \quad |a_3|^2 \approx 94\%$